Tuesday, December 1, 2015

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Problem 38

Problem. Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$ converges absolutely or conditionally, or diverges.

Solution. As a p-series, the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges. Therefore, $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$ converges absolutely.

Problem 39

Problem. Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n}}{n!}$ converges absolutely or conditionally, or diverges.

Solution. By the Direct Comparison Test or the Limit Comparison Test with $\displaystyle\sum_{n=0}^{\infty} \frac{1}{2^n}$, the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n!}$ converges. Therefore, $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n}}{n!}$ converges absolutely.

Problem 40

Problem. Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+3}$ converges absolutely or conditionally, or diverges.

Solution. By the Alternating Series Test, $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+3}$ converges. However, by the Limit Comparison Test with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$, the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+3}$ diverges. Therefore, $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+3}$ converges conditionally.

Problem 41

Problem. Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ converges absolutely or conditionally, or diverges.

Solution. By the Alternating Series Test, $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ converges. However, the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges. Therefore, $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ converges conditionally.

Problem 42

Problem. Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\sqrt{n}}$ converges absolutely or conditionally, or diverges.

Solution. By the Alternating Series Test, $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\sqrt{n}}$ converges. In addition, as a p-series, the series $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$ also converges. Therefore, $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ converges absolutely.

Problem 43

Problem. Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}n^2}{(n+1)^2}$ converges absolutely or conditionally, or diverges.

Solution. By the Divergence Test, $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}n^2}{(n+1)^2}$ diverges because $\lim_{n\to\infty} \frac{n^2}{(n+1)^2} \neq 0$.

Problem 45

Problem. Determine whether the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln n}$ converges absolutely or conditionally, or diverges.

Solution. By the Alternating Series Test, $\sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln n}$ converges. By the Integral Test, $\sum_{n=2}^{\infty} \frac{1}{n\ln n}$ diverges. Therefore, $\sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln n}$ converges conditionally.

Problem 71

Problem. Test the series $\sum_{n=1}^{\infty} \frac{10}{n^{3/2}}$ for convergence or divergence and identify the test used.

Solution. This series is 10 times the p-series $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$, which converges. Therefore, by the p-series test, $\sum_{n=1}^{\infty} \frac{10}{n^{3/2}}$ converges.

Problem 73

Problem. Test the series $\sum_{n=1}^{\infty} \frac{3^n}{n^2}$ for convergence or divergence and identify the test used.

Solution. Use the Divergence Test and L'Hôpital's Rule.

$$\begin{aligned}\lim_{n\to\infty} \frac{3^n}{n^2} &= \lim_{n\to\infty} \frac{3^n \ln 3}{2n}\\ &= \lim_{n\to\infty} \frac{3^n (\ln 3)^2}{2}\\ &= \infty.\end{aligned}$$

Therefore, $\sum_{n=1}^{\infty} \frac{3^n}{n^2}$ diverges.

Problem 75

Problem. Test the series $\sum_{n=0}^{\infty} 5\left(\frac{7}{8}\right)^n$ for convergence or divergence and identify the test used.

Solution. This series is 5 times the geometric series $\sum_{n=0}^{\infty} \left(\frac{7}{8}\right)^n$, which converges because $\frac{7}{8} < 1$. Therefore, $\sum_{n=0}^{\infty} 5\left(\frac{7}{8}\right)^n$ converges.

Problem 78

Problem. Test the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+4}$ for convergence or divergence and identify the test used.

Solution. By the Alternating Series Test, $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+4}$ converges.

Problem 80

Problem. Test the series $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n}$ for convergence or divergence and identify the test used.

Solution. Use the Direct Comparison Test, comparing to $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n}$, which diverges. Obviously, $\dfrac{\ln n}{n} > \dfrac{1}{n}$ for all $n > e$. Therefore, $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n}$ diverges.